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SO(2,C) invariant discrete gauge states in liouville gravity coupled to minimal conformal matter

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Abstract. We construct the general formula for a set of discrete gauge states (DGS) in c < 1 Liouville theory. This formula reproduces the previously found c = 1 DGS in the appropriate limiting case. We also demonstrate the SO(2,C) invariant structure of these DGS in the old covariant quantization of the theory. This is in analogy to the SO(2,C) invariant ring structure of BRST cohomology of the theory.

I Introduction

Liouville gravity [1] has been an important consistency check of the discretized matrix model approach to non-perturbative string theory for the last few years. Many interesting phenomena, which were peculiar to 2D string, were uncovered and attempts have been made to compare these results to the more realistic high dimensional string theory [2]. There are two quantization schemes of the theory which appeared in the liturature. In the most popular BRST approach, the ground ring structure of ghost number zero operators accounts for the existence of space-time w_∞ symmetry of the theory [3]. In the old covariant quantization (OCQ) scheme, we believe that the gauge states [4] (physical zero-norm states) will play the role of nontrivial ghost sector in BRST approach. However, unlike the discrete Polyakov states, there is an infinite number of continuum momentum gauge states in the massive levels of the spectrum and it is difficult to give a general formula for them. In previous papers [2, 4], we introduced the concept of DGS (gauge states with Polyakov discrete momentum) in the OCQ scheme and explicitly constructed a set of them for c = 1 Liouville theory. We then showed that they do carry the w_∞ charges. An explicit form of worldsheet supersymmetric DGS in N=1 superLiouville theory was also given in [5].

Since the idea of gauge states has direct application in the high dimensional string theory [2], it would be important to understand it more in general Liouville theory and compare it with the known results in BRST Liouville theory. In this paper, we will show that the SO(2,C) invariant ring structure of BRST cohomology of the c ≤ 1 Liouville model [6] has its counterpart in OCQ scheme, that is *the SO(2, C) invariant DGS* . We will first construct the general formula for a set of DGS in c < 1 Liouville theory. This formula reduces to the c = 1 DGS in the p = q + 1, q→ ∞ limit. We then show that the particular set of DGS we constructed together with the c = 1 DGS discovered previously [4] form a SO(2,C) invariant set. It is thus easily seen that this c < 1 DGS also carrys the w_∞ charges.

II Discrete gauge states in c < 1 Liouville theory

We consider the following action of c < 1 Liouville theory [7]

$$S = \frac{1}{8\pi}\int d^2z\sqrt{\hat{g}}\;\left[\hat{g}^{\alpha\beta}\partial_\alpha X\partial_\beta X + 2iQ_M\hat{R}X + \hat{g}^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + 2Q_L\hat{R}\phi\right], \quad (2.1)$$

with ϕ being the Liouville field. If we take

$$Q_M = (p-q)Q, \quad Q_L = (p+q)Q \quad (2.2)$$

with $Q = \frac{1}{\sqrt{2pq}}$ and p,q are two coprime positive integers, the central charges for both fields will be

$$c_M = 1 - \frac{6(p-q)^2}{pq}, \quad c_L = 1 - \frac{6(p+q)^2}{pq} \quad (2.3)$$

so that

$$c_M + c_L = 26 \quad (2.4)$$

which cancels the anomaly from ghost contribution. The stress energy tensor is (from now on we consider the chiral sector only)

$$T_{zz} = -\frac{1}{2}(\partial X)^2 + iQ_M\partial^2 X - \frac{1}{2}(\partial\phi)^2 - Q_L\partial^2\phi. \quad (2.5)$$

Note that if we take the p = q + 1, q→ ∞ limit, we recover the usual unitary $c_M = 1$ 2D gravity model. The mode expansion of $X^\mu = (\phi, X)$ is defined to be

$$\partial_z X^\mu = -\sum_{n=-\infty}^{\infty} z^{-n-1}(\alpha_n^0, i\alpha_n^1), \quad (2.6)$$

with the metric $\eta_{\mu\nu} = diag[-1,1], Q^\mu = (iQ_L, Q_M)$ and the zero mode $\alpha_0^\mu = f^\mu = (\epsilon, p)$. The corresponding Virasoro generators are

$$L_n = (\frac{n+1}{2}Q^\mu + f^\mu)\alpha_{\mu,n} + \frac{1}{2}\sum_{k\neq 0} : \alpha_{\mu,-k}\alpha^\mu_{n+k} : \ n \neq 0, \tag{2.7}$$

$$L_0 = \frac{1}{2}(Q^\mu + f^\mu)f^\mu + \sum_{k=1}^{\infty} : \alpha_{\mu,-k}\alpha^\mu_k : . \tag{2.8}$$

In the OCQ scheme, physical states $|\psi>$ are those satisfy the condition

$$L_n|\psi>= 0 \ \ for\ n > 0, \quad L_0|\psi>= |\psi> . \tag{2.9}$$

The massless tachyon

$$O_j = e^{i\beta_j X + \alpha_j \phi} \tag{2.10}$$

are positive norm physical states if either of the on-shell condition

$$\pm(\beta_j - Q_M) = (\alpha_j + Q_L) \tag{2.11}$$

is satisfied. If one defines

$$\beta_j = jQ + (p-q)Q, \tag{2.12}$$

then

$$\alpha_j^\pm = \pm|jQ| - (p+q)Q. \tag{2.13}$$

It's now easy to see that

$$O^+_{2q} = e^{i(p+q)QX - (p-q)Q\phi}, \quad O^+_{-2p} = e^{-i(p+q)QX + (p-q)Q\phi} \tag{2.14}$$

together with

$$\int \frac{dz}{2\pi i} O^+_{-2p}(z) O^+_{2q}(0) \sim \partial_z[i(p+q)QX - (p-q)Q\phi] \tag{2.15}$$

are the zero modes of generators of the level one $SU(2)_{k=1}$ Kac-Moody algebra. Note that there is no concept of “material gauge” as one has in the c=1 theory and the Liouville field ϕ appears in (2.15). In general there exist discrete states ($J = \{0, \frac{1}{2}, 1, ...\}$ and M= { -J,-J+1,...,J})

$$\Psi^\pm_{J,M} \sim \left(\int \frac{dz}{2\pi i} O^+_{-2p}(z)\right)^{J-M} O^\pm_{2Jq}(0) \tag{2.16}$$

One can express the discrete states in (2.16) in terms of Schur polynomials, which are defined as follows:

$$exp\left(\sum_{k=1}^\infty a_k x^k\right) = \sum_{k=0}^\infty S_k(a_k)x^k, \tag{2.17}$$

where S_k is the Schur polynomial, a function of $\{a_k\} = \{a_i : i \in Z_k\}$. An explicit calculation of (2.16) gives

$$\Psi^\pm_{J,M} \sim \begin{vmatrix} S_{2J-1} & S_{2J-2} & \cdots & S_{J+M} \\ S_{2J-2} & S_{2J-3} & \cdots & S_{J+M-1} \\ \vdots & \vdots & \ddots & \vdots \\ S_{J+M} & S_{J+M-1} & \cdots & S_{2M+1} \end{vmatrix} \times \exp\left(i\left[(\pm J + M - 1)q - (\pm J - M - 1)p\right]QX + \left[(\pm J + M - 1)q + (\pm J - M - 1)p\right]Q\phi\right) \tag{2.18}$$

with $S_k = S_k(\{\frac{1}{k!}\partial^k[-iQ_L X + Q_M\phi]\})$ and $S_k = 0$ if $k < 0$. We will denote the determinant in (2.18) as $\triangle(J, M, -iQ_L X + Q_M\phi)$.

In the OCQ of the theory, in addition to the positive norm physical states as discussed above, we still have an infinite number of *continuum momentum* gauge states in the spectrum. They are solutions of either of the following equations:

$$|\psi>= L_{-1}|\chi> \quad \text{where } L_m|\chi>= 0 \ \ m \geq 0, \tag{2.19}$$

$$|\psi>= (L_{-2} + \frac{3}{2}L^2_{-1})|\xi> \quad \text{where } L_m|\xi>= 0 \ \ m > 0, \quad (L_0 + 1)|\xi>= 0. \tag{2.20}$$

They satisfy the physical states conditions (2.9), and have zero norm. Note that (2.20) is a gauge state only when the critical condition (2.4) is satisfied. It's difficult to give the general formula for all the solutions of (2.19) and (2.20). However, as was motivated from the c=1 theory [4], we propose the following DGS for the Ψ^- sector

$$\begin{aligned} G^-_{J,M} &\sim \left[\int \frac{dz}{2\pi i} O_0(z)\right] \Psi^-_{J-1,M}(0) \\ &\sim \left[\int \frac{dz}{2\pi i} e^{i(p-q)QX - (p+q)Q\phi}(z)\right] \Psi^-_{J-1,M}(0) \\ &\sim S_{2J-1}(\{\frac{-1}{k!}\partial^k[(q+p)Q\phi + i(q-p)QX]\}) \\ &\quad \triangle(J-1, M, -iQ_L X + Q_M\phi) \\ &\quad \exp\left(i\left[(-J+M-1)q + (J+M+1)p\right]QX + \left[(-J+M-1)q - (J+M+1)p\right]Q\phi\right). \end{aligned} \tag{2.21}$$

It can be proved that these states are zero norm states and satisfy the physical states condition (2.9). As an example, for $J = \frac{3}{2}, M = \pm\frac{1}{2}$

$$\begin{aligned} G^-_{\frac{3}{2},\pm\frac{1}{2}} &= \left[Q_L^2(\partial\phi)^2 - 2iQ_L Q_M \partial\phi\partial X - Q_M^2(\partial X)^2 - Q_L\partial^2\phi + iQ_M\partial^2 X\right] \\ &\quad \times e^{(\mp\frac{1}{2}Q_M - \frac{5}{2}Q_L)\phi + (\pm\frac{i}{2}Q_L + i\frac{5}{2}Q_M)X}, \end{aligned} \tag{2.22}$$

which can be shown to be a mixture of solutions of (2.19) and (2.20). For the Ψ^+ sector, we can subtract two positive norm discrete states as was done in the c=1 theory [4] to obtain a gauge state:

$$G^+_{J.M} \sim \int \frac{dz}{2\pi i}[\Psi^+_{1,-1}(z)\Psi^+_{J,M+1}(0) + \Psi^+_{J,M+1}(z)\Psi^+_{1,-1}(0)]. \tag{2.23}$$

As an example, we have

$$G^{-}_{\frac{3}{2},\pm\frac{1}{2}} = \left[\frac{1}{4}\begin{pmatrix} -Q_L^2+5Q_M^2\pm 6iQ_LQ_M & \pm 3Q_L^2\pm 3Q_M^2-4iQ_LQ_M \\ \pm 3Q_L^2\pm 3Q_M^2-4iQ_LQ_M & -5Q_L^2+Q_M^2\mp 6iQ_LQ_M \end{pmatrix} \times\partial X^\mu\partial X^\nu - i\frac{1}{2}\begin{pmatrix} Q_L\mp 5iQ_M \\ -iQ_M\mp 5Q_L \end{pmatrix}\partial X^\mu\right] \times e^{\frac{1}{2\sqrt{2}}(\mp Q_M+Q_L)\phi+\frac{1}{2\sqrt{2}}(\pm iQ_L-iQ_M)X}, \qquad (2.24)$$

which can be shown to be solution of (2.20).

III SO(2,C) invariant and w_∞ charges

It can be easily seen that (2.18), (2.21) and (2.23) reduce to similar equations in c=1 theory when we take the $p = q+1, q = \infty$ limit. On the other hand, the DGS in (2.21) and (2.23) can be obtained by doing a SO(2,C) rotation on that of the c=1 theory

$$M = \frac{Q}{\sqrt{2}}\begin{pmatrix} q+p & i(q-p) \\ -i(q-p) & q+p \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} Q_L & -iQ_M \\ iQ_M & Q_L \end{pmatrix} \qquad (3.1)$$

where $M \in SO(2,C)$. A similar result was noticed in the ring structure of BRST cohomology in [6]. The SO(2,C) invariant structure of the DGS in the OCQ scheme suggests that the gauge state equations (2.19) and (2.20) may possess a hidden symmetry property which has implication on all the continuum momentum gauge states.Our results in this paper show once again that the structure of DGS in the OCQ scheme is closely related to the nontrivial ghost sector in the BRST quantization of the theory. Finally, because of SO(2,C) invariance, from (2.23) and a similar argument to the c=1 theory, one easily see that the c< 1 DGS constructed here also carrys the w_∞ charges.

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